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Thought Experiments at Superluminal Relative Velocities

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Abstract

It is imagined that our world is being examined from a similar world which is moving relative to us with a velocity greater than that of light. The two worlds are supposed to be similar in that the particles in each appear to any observer in that world to have real measurable properties. However, the enormous relative velocity so distorts the observations that each world makes on the other that the squares of certain real quantities appear to the other observer to be negative. Neglect of this fact has led to the erroneous belief that a free charged tachyon would emit Čerenkov radiation and that the existence of tachyons would lead to logical paradoxes.

In recent years, two entirely different viewpoints about tachyons have emerged,[1] and with characteristic scientific detachment, the proponents of each side believe that members of the other are completely wrong. The first and orthodox viewpoint (Bilaniuk et al., 1962; Feinberg, 1967; Arons and Sudarshan, 1968; Dhar and Sudarshan, 1968; Bilaniuk and Sudarshan, 1969) is based on the seemingly sound assumption that all observations, including those we might make on tachyons, must lead to real quantities. Admittedly, the rest mass of a tachyon appears from this viewpoint to be imaginary, but since the particle cannot be brought to rest this is not regarded as significant. However, it is assumed that momenta, positions, angular momenta, and other dynamical variables that we may ascribe to a tachyon must be real. Despite this (though actually because of this) tachyons emerge from this point of view as objects that are basically very different from those with which we are familiar in our world of bradyons; e.g., if charged they would emit Čerenkov radiation spontaneously, and their existence would lead to serious logical

[1] We neglect here the majority viewpoint, that the question is not worth pursuing.

paradoxes. The question of the complex quantities that automatically appear in superluminal transformations is usually (cf., however, Antippa and Everett, 1971; Antippa, 1975) dodged by avoiding such transformations altogether, i.e., by looking at the world only from our viewpoint, and not that of an observer riding on a tachyon. In a sense it is an egocentric view of the problem.

The second viewpoint[2] notes that a number of objects that are tachyons relative to us would be bradyons relative to each other, and therefore imagines a hypothetical world of tachyon matter, no different from the world as we know it, but distinguished only by the fact that, *relative to us*, it has a superluminal velocity. All measurements we make of our world of objects that are bradyons relative to us must be real, and all measurements made by an observer in this other world of objects that are bradyons relative to him must, also be real. This is therefore a much more symmetrical viewpoint, which seems to be more in accordance with the principle of relativity, but it means that when we make observations on a tachyon some of the physical quantities we attempt to measure will not in fact be measurable, and formally they will appear in the analysis as imaginary quantities. This result is unacceptable to the protagonists of the egocentric viewpoint, for whom all measurements *we* make must be real, whatever the tachyon observer may measure.

The disagreement may be analyzed in terms of the measurements made by two observers O, O' of the length of a rod held by O' in a direction orthogonal to their mutual velocity $v = \beta c$ in the z direction. By sending a flash of light to the other end of the rod, O' measures the length l' as a very real quantity $l' = ct'$, where t' is the time of transit for the light flash, as measured by O'. In looking at O's view of the measurement, it is usual to assume that OO were coincident with $z = 0$ at the moment the light was flashed, and that O observes the light to reach the other end of the rod at $z = vt$, after it has traversed a distance ct. The length l of the rod is then calculated by O to be $t(c^2 - v^2)^{1/2}$ and since for $v < c$, $t' = t(1 - \beta^2)^{1/2}$ it follows that $l = ct' = l'$. For $v > c$, however, the length of the rod as measured by O is formally imaginary because a ray of light sent along the rod in the frame of O' never catches up to the end of the rod in the frame of O.

Now this argument is based on the assumption that the velocity of light is the same for both observers, but that assumption implies that some light signals emitted from the point O' at the time O' and O are coincident can never be measured by O himself. A beam directed by O' to a point P at rest with respect to him such that $O'P$ is at right angles to the relative velocity of O and O' can never be seen by O. There is simply no way for either observer to send a light signal between the ends of a rod held by the other in the plane normal to their relative velocity. This is the physical reason why mathematically it has been known for some time that superluminal transformations are possible only within the complex Lorentz group (Pauli, 1921; Gorini, 1971).

[2] Represented by Balbo et al. (1970), Olkhovsky and Recami (1971), Recami and Mignani (1972, 1974), Mignani and Recami (1973, 1974), Yaccarini (1974), and Corben (1974).

Of course, it can always be argued that a tachyon might leave a track in a bubble chamber, and its coordinates normal to its path could therefore be measured. In principle this is correct, but then an observer riding on the tachyon would be unable to measure those coordinates. If the measurement is real for one observer it cannot be made by the other.

In the absence of experimental data, the difference between the two attitudes borders on that time-honored polarizer of ideas, metaphysics. However, the debate is worth pursuing because the experimental consequences would be entirely different if tachyons were found to exist. Above all, from the second point of view, as pointed out by the authors cited in footnote 2 above, a free charged tachyon could not emit Čerenkov radiation or any other particle. Since the world of tachyons is supposed in this model to be identical with the world as we know it, no logical paradoxes could arise from the existence of tachyons.

This paper is therefore concerned with a thought experiment within the framework of this latter viewpoint. We imagine an observer, O', equipped with the apparatus for making measurements with which we are all familiar, moving relative to us with a constant velocity that is greater than that of light. We refer our own observations to a coordinate system O, with respect to which our velocity is small compared with that of light, and we wish to compare these observations with those made by O'.

As noted, we assume from the outset that all physical quantities that we measure in our world are real, i.e., particles that appear to us to be bradyons have real positions, velocities, momenta, masses, etc. and generate real electromagnetic fields. By the same token, we assume that the same holds true for O'—particles that are bradyons from the point of view of O' have real properties when measured in the coordinate system of O'. We therefore assume a basic symmetry between O and O'—two real worlds trying to understand what happens when they make measurements on each other. In this sense our picture is the same as that developed by Recami and Mignani (1972, 1974), although we differ from them in a number of details.

We demonstrate here that this thought experiment automatically allows us to avoid a number of paradoxes in tachyon theory. However, the view O' would have of our world would be so distorted by his enormous velocity relative to it that certain physical quantities, which appear real enough to us, would to him appear imaginary—he would be unable to make a direct measurement of them. By the principle of relativity the same is true in reverse—if we attempt to observe his world, similar problems would arise. Each would infer that the squares of some quantities that he tries to measure in the world of the other are negative.

The concept that a tachyon would have an imaginary rest mass has been exploited in a search for particles with negative "missing mass squared" in elementary particle collisions (Baltay et al., 1970; Dansburg et al., 1971), so the introduction of imaginary physical quantities into tachyon theory is not new. Indeed, the postulate that in the framework of O' a bradyon would have a real rest mass m_0 leads to the consequence that in our system it would be a tachyon whose energy and momentum are related by

$$p_\sigma p^\sigma \equiv W^2 - p^2c^2 = -m_0{}^2c^4 = -p'_\sigma p'^\sigma$$

Our distorted view of a particle in the world of O' ascribes to that particle the appearance of having an imaginary rest mass, whereas in fact its rest mass is real (see references cited in footnote 2 above).

We assume that, according to the principle of relativity, the velocity of light is the same for all observers, so that $dr^2 = c^2dt^2$ (as measured by O) implies that $dr'^2 = c^2dt'^2$ (as measured by O'). Thus

$$dx_\sigma dx^\sigma = A dx'_\sigma dx'^\sigma \tag{1}$$

Although A is arbitrary, if we wish to avoid unnecessary scale changes we require $A = \pm 1$. If further

$$p_\sigma p^\sigma = -p'_\sigma p'^\sigma$$

and p_σ, dx_σ transform in the same way, it follows that $A = -1$.

If the velocity of O' relative to O is $\mathbf{v}$, where $v = |\mathbf{v}| = \beta c > c$, the transformation that corresponds to $A = -1$ is

$$\begin{aligned} d\mathbf{r}' &= -i d\mathbf{r} + (\Gamma + i)\frac{\mathbf{v}\cdot d\mathbf{r}}{v^2}\mathbf{v} - \Gamma\mathbf{v}dt \\ dt' &= \Gamma\left(dt - \frac{\mathbf{v}\cdot d\mathbf{r}}{c^2}\right) \end{aligned} \tag{2}$$

where $\Gamma = \pm(\beta^2 - 1)^{-1/2}$, so that

$$\begin{aligned} d\mathbf{r} &= i d\mathbf{r}' - (\Gamma + i)\frac{\mathbf{v}\cdot d\mathbf{r}'}{v^2}\mathbf{v} - \Gamma\mathbf{v}dt' \\ dt &= -\Gamma\left(dt' + \frac{\mathbf{v}\cdot d\mathbf{r}'}{c^2}\right) \end{aligned} \tag{3}$$

[The case $A = +1$ is obtained by multiplying the right-hand sides by i, $-i$, thus making time intervals and longitudinal distances appear imaginary to O'.] If $\mathbf{v}$ is in the common z direction, the above equations become (see references cited in footnote 2 above)

$$\begin{aligned} dz' &= \Gamma(dz - v\,dt), \qquad & dx' &= -i\,dx \\ dt' &= \Gamma[dt - (v/c^2)dz], \qquad & dy' &= -i\,dy \end{aligned} \tag{2'}$$

or

$$\begin{aligned} dz &= -\Gamma(dz' + v\,dt'), \qquad & dx &= i\,dx' \\ dt &= -\Gamma[dt' + (v/c^2)dz'], \qquad & dy &= i\,dy' \end{aligned} \tag{3'}$$

These are clearly very similar to the equations that describe the Lorentz transformation for subluminal relative velocities, except for the appearance of imaginary quantities associated with directions orthogonal to the superluminal relative velocity **v**. A number of attempts have been made (Antippa and Everett, 1971; Antippa, 1975) to avoid these imaginary and hence (apparently) meaningless quantities, but it is not possible to do so and remain consistent with the principle of relativity and its consequence, equation (1).

We first consider the real quantities involved in the transformation laws. According to equations (2) and (3) or (2′) and (3′) time, and indeed the zeroth component of any four-vector, may appear contracted or expanded, but it always appears real. For a clock at rest in "our" frame O,

$$dt' = \Gamma dt \tag{4}$$

where $\Gamma = (\beta^2 - 1)^{-1/2}$ exceeds unity for $\beta < \sqrt{2}$, but is less than unity for $\beta > \sqrt{2}$. A reciprocal relation would apply if we attempted to observe a clock in O', i.e., $dt = -\Gamma dt'$. In addition to consequences similar to the usual time dilatation phenomenon, the negative sign that appears here shows that O' would think our clocks were moving forward but we would think that clocks carried by O' were moving backwards. This may be seen from Figure 1, in which O' sends off a light signal to us at uniform intervals in his frame. If at a certain distance O' turns around and returns with the same speed as before, we note from Figure 1 that his redshift $1 + z_r = [(\beta + 1)/(\beta - 1)]^{1/2}$ now becomes a blueshift $1 + z_b = [(\beta - 1)/(\beta + 1)]^{1/2}$ and that the "twin paradox" is resolved in the usual manner. It is apparent from the figure that there is no indication of any imaginary time intervals, such as would have arisen had we chosen $A = 1$ in equation (1). It is also apparent that as long as O' is receding from O, the light signals that he emits are received by O in the order in which they

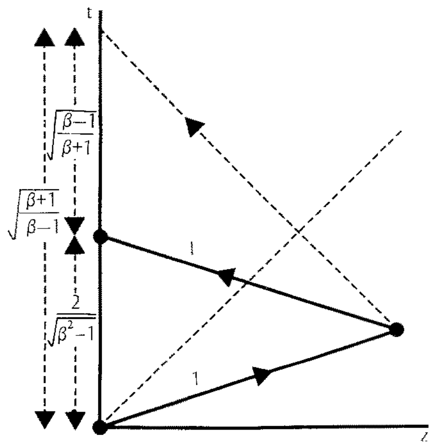

Figure 1. "Twin paradox" at superluminal relative velocities.

are transmitted, whereas, when O' appears to O to be approaching, O receives O's light signals in the reverse order. From the point of view of O, the total time taken for O's journey is then

$$\left(\frac{\beta+1}{\beta-1}\right)^{1/2} - \left(\frac{\beta-1}{\beta+1}\right)^{1/2} = 2\Gamma$$

and from O's point of view, the journey occupied 2 units of time, giving the ratio Γ of equation (4).

The relative signs of time intervals may also be seen from Figure 2. Signals 1 and 2 sent from O reach O' in the order in which they were sent, so that O' measures O's clock as moving forward. On reflection back to O, however, these signals are received in reverse order, so that O measures O''s clock as moving backward. For signals a, b, however, O' measures O's clock as moving backwards, but O measures O''s clock as moving forward. These two cases correspond to the two possible signs of Γ in equations (2) and (3). Similarly, the rod $A'B'$ held in O' is measured by O as the length AB' which is in the negative z direction if $A'B'$ is in the positive z' direction.

We now return to the appearance of imaginary quantities in the transformation laws. It has been noted that if O' directs a flash of light to a point in his frame at right angles to the relative motion, O will never see it and

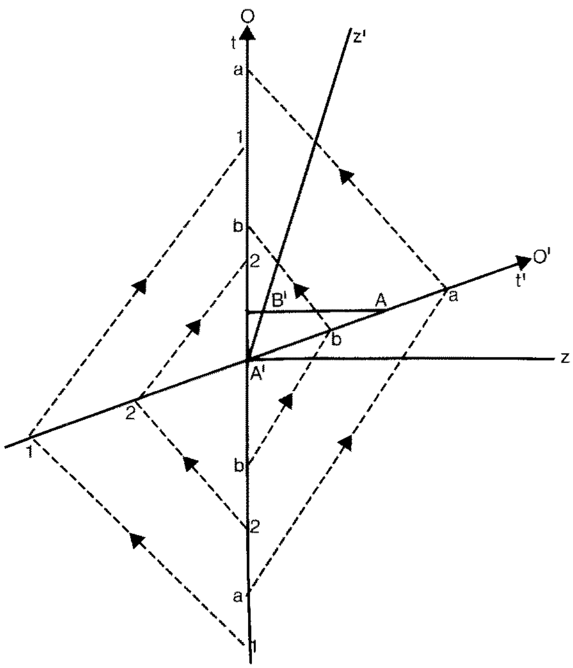

Figure 2. Light signals sent from O to O' and reflected back to O return in the reverse order.

therefore cannot use it to measure in his frame transverse lengths in the frame of O'. Thus if O' measures a transverse length as a real quantity O cannot, formally ascribing to it an imaginary value, and vice-versa. Indeed it is clear from equation (2) that it is not possible for a superluminal transformation to lie within the real Lorentz group unless

$$\frac{\mathbf{v}\cdot d\mathbf{r}}{v^2}\mathbf{v} = d\mathbf{r}$$

i.e., unless consideration is restricted to distances in the direction of the relative velocity $\mathbf{v}$ (Parker, 1969). In general, writing $(\mathbf{v}\cdot d\mathbf{r})/v = d\epsilon = \cos\theta\, dr$, we have

$$\begin{aligned} dr'^2 &= d\epsilon^2 - dr^2 + \Gamma^2(d\epsilon - \beta c dt)^2 \\ &= d\epsilon^2 - dr^2 + \Gamma^{-2}(d\epsilon + \Gamma\beta c dt')^2 \end{aligned} \tag{5}$$

Thus a rod held in O at the angle θ to the direction of the relative velocity will have a real length dr' relative to O' if $|\cos\theta| > 1/\beta$.

We note that this condition $|\cos\theta| > 1/\beta$ for the measurability by O' of the length of a rod held by O may be expressed in terms of measurements made in O' according to the conditions

$$dt' = 0, \qquad \frac{\mathbf{v}\cdot d\mathbf{r}'}{v} = \cos\theta'\, dr'$$

$$\mathbf{v}\cdot d\mathbf{r}' = -\Gamma^{-1}(\mathbf{v}\cdot d\mathbf{r})$$

so that

$$dr'^2 = (\beta^2\cos^2\theta - 1)dr^2$$

may be written

$$dr^2 = \left(\frac{\beta^2}{\beta^2 - 1}\cos^2\theta' - 1\right)dr'^2$$

Thus dr' is real if $|\cos\theta'| > (\beta^2 - 1)^{1/2}/\beta$ or $|\sin\theta'| < 1/\beta$ which defines the "Čerenkov" cone (Thomson, 1889) as observed by O'. Only lengths that lie inside this cone can be measured by O'. This result may be understood physically by noting that if the rod held by O appears to O' to be held at a sufficiently small angle to the direction of the relative motion, a signal sent from the leading end of the rod to the trailing end could in fact be observed by O', although O' could not observe it if it were transmitted in the opposite sense. The maximum angle as seen by O' that would permit this is easily seen to be given by $|\sin\theta'| = 1/\beta$. Naturally there is a similar relation for the observation by O of a rod held in O'.

We could of course imagine a situation in which O' observes real flashes of light emanating successively from $x' = 0, y' = 0, z' = 0, t' = 0$, and then $x' = 0$,

$y' = y_0$, $z' = -vt_0$, $t' = t_0$, where all quantities are real, and $v > c$. In this case, if the events are causally related, they must have been caused not by a ray of light but by a tachyon moving with velocity $U = (v^2 + y_0^2/t_0^2)^{1/2}$ relative to O'. Relative to O this tachyon would be a bradyon with velocity βc in the y direction relative to O where

$$\beta^2 = \frac{-y_0^2}{c^2 t_0^2(\beta^2 - 1)}$$

Thus if, as postulated, y_0 is real and $\beta > 1$, this represents an unreal situation in the frame of O and is inconsistent with our postulate. Since $\beta = y/ct$ and $t = t_0(\beta^2 - 1)^{1/2}$ we are again led to the relation

$$y^2 = -y'^2$$

If on the other hand a real situation exists in the frame of O, who measures the real distance y by emitting a particle with velocity $v < c$ and noting that it takes time t to reach y, the pattern observed by O' would not be that described above, for from his point of view the particle, although a tachyon, would have a velocity $U < v$, resulting in his inability to measure both its emission and its arrival at y.

We emphasize that these unusual consequences appear only because of the very distorted views of each other's world that O and O' acquire because of their superluminal relative velocity. All lengths and masses in the world of O are real relative to an observer in O, and all lengths and masses in the world of O' are equally real to an observer in O'. If, however, we imagine one observer to examine an object (which he calls a tachyon) from the world of the other we are forced to the conclusion that he is unable to measure some of the components of position and velocity of this object, and that these components then appear in his equations as imaginary quantities. The common assumption that they would be real is to assert that in its own world the object would have an imaginary position coordinate, and is clearly opposed to the basic postulate of our thought experiment, and is difficult if not impossible, to interpret physically.

A prime example of this error is the belief that a free charged tachyon would emit Čerenkov radiation (Alväger and Kreisler, 1968; Davis et al., 1969) if $E_i = cp_i \cos\theta$, $E_f = cp_f \cos(\theta + \phi)$ [$(E_i, \mathbf{p}_i)$ initial and $(E_f \mathbf{p}_f)$ final energies and momenta of the tachyon, ϕ = scattering angle of tachyon, θ = angle of photon emission]. By assuming that this is a real process as observed in our system, we are forced to the conclusion that in the system in which the initial tachyon is at rest the emitted photon has a longitudinal component of momentum $p' = p_f \sin\phi$ and a transverse component ip'. Čerenkov radiation in our system O would look like this imaginary and unphysical process in the system O' with respect to which the tachyon is at rest, and is therefore contrary to our initial assumption. *Only real processes should be admitted to objects that appear to the observer to be bradyons.* With this assumption Čerenkov radiation of a free tachyon could not occur, and the fact that it has not been observed is of no significance.

A similar argument applies to the Pirani (1970) paradox, for if the physical situation described by this paradox is transformed to the system in which one of the tachyons is at rest, one is forced to imaginary quantities in that system because of the two-dimensional nature of the problem. If, however, four real bradyons in O' are allowed to move in any two-dimensional circuit, transformation to the system O in which they are tachyons would automatically introduce masses and some lengths the squares of which would appear to us to be negative. This would represent O's (our) distorted view of a very simple picture, but it would not represent the physical situation described by the Pirani paradox and would not lead to any other paradoxical situation. Once again the problem lies in the assumption that tachyons always have real properties when measured in our coordinate systems, whereas all of their properties are real only in the world in which they are bradyons.

In conclusion, it must be noted that despite (or really because of) the imaginary quantities that appear in superluminal Lorentz transformations, the velocity of light for all such observers is real, and has the same value for all. However, lengths in one frame that lie along the Čerenkov cone as seen by the other are zero, and lengths that lie outside of that cone cannot be measured by the other.

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References

Alväger, T., and Kreisler, M. N. (1968). *Physical Review*, **171**, 1357.
Antippa, A. F. (1975). *Physical Review D*, **11**, 724.
Antippa, A. F., and Everett, A. E. (1971). *Physical Review D*, **4**, 2198.
Arons, M. E., and Sudarshan, E. C. G. (1968). *Physical Review*, **173**, 1622.
Balbo, M., Fonte, G., and Recami, E. (1970). *Lettere al Nuovo Cimento*, **4**, 241.
Baltay, C., Feinberg, G., Yeh, N., and Linsker, R. (1970). *Physical Review D*, **1**, 759.
Bilaniuk, O. M., Deshpande, V. K., and Sudarshan, E. C. G. (1962). *American Journal of Physics*, **30**, 718.
Bilaniuk, O. M., and Sudarshan, E. C. G. (1969). *Physics Today*, **22**, No. 5, 43.
Corben, H. C. (1974). *Lettere al Nuovo Cimento*, **11**, 533.
Dansburg, J. S., Kalbfleisch, G. R., Borenstein, S. R., Strand, R. C., Vanderburg, V., Chapman, J. W., and Lys, J. (1971). *Physical Review D*, **4**, 53.
Davis, M. B., Kreisler, M. N., and Alväger, T. (1969). *Physical Review*, **183**, 1132.
Dhar, J., and Sudarshan, E. C. G. (1968). *Physical Review*, **174**, 1808.
Feinberg, G. (1967). *Physical Review*, **159**, 1089.
Gorini, V. (1971). *Communications in Mathematical Physics*, **21**, 150.
Mignani, R., and Recami, E. (1973). *Nuovo Cimento*, **14A**, 169.
Mignani, R., and Recami, E. (1974). *Lettere al Nuovo Cimento*, **9**, 354.
Olkhovsky, V. S., and Recami, E. (1971). *Lettere al Nuovo Cimento*, **1**, 165.
Parker, L. (1969). *Physical Review*, **188**, 2287.
Pauli, W. (1921). *Relativitätstheorie*. (B. G. Teubner, Leipzig).
Pirani, F. A. E. (1970). *Physical Review D*, **1**, 3224.

Recami, E., and Mignani, R. (1972). *Lettere al Nuovo Cimento*, **4**, 144.
Recami, E., and Mignani, R. (1974). *Revista del Nuovo Cimento*. **4**, 209. (and many references contained therein).
Thomson, J. J. (1889). *Philosophical Magazine*, **28**, 13.
Yaccarini, A. (1974). *Lettere al Nuovo Cimento*, **9**, 354.